HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

2013

Trial HSC

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General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Attempt **all** questions.
- Board approved calculators and Math Aids may be used.
- This examination must **NOT** be removed from the examination room

- **Section A** consists of ten (10) multiple choice questions worth 1 mark each. Fill in your answer on the multiple choice answer sheet provided.
- **Section B** requires all necessary working to be shown in every question. This section consists of four (4) questions worth 15 marks each. Marks may not be awarded for careless or badly arranged work.
  Each question is to be started in a new answer booklet. Additional booklets are available if required.

Name : ________________________

Class : __________

SECTION A – 10 multiple choice questions (1 mark each)

The answer sheet may be torn off the back of the exam

Question 1

Ten kilograms of chlorine is placed in water and begins to dissolve.

After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$.

What is the value of k given that $A = 3.6$ and $t = 5$?

(A) –0.717 (B) –0.204 (C) 0.204 (D) 0.717

Question 2

Consider the polynomial $P(x) = 3x^3 + 3x + a$.
If $x - 2$ is a factor of $P(x)$, what is the value of a?

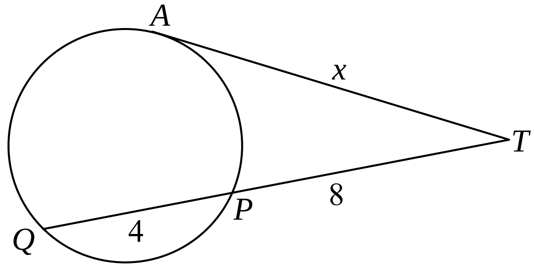
(A) –30 (B) -18 (C) 18 (D) 30

Question 3

$\tan^{-1}(-1) =$

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Question 4



NOT TO SCALE

In the above diagram, TA is a tangent, QP is a chord produced to T. What is the value of x?

(A) 12 (B) $2\sqrt{3}$ (C) $4\sqrt{2}$ (D) $4\sqrt{6}$

Question 5

A flat circular disc is being heated so that the rate of increase of the area (A in m^2), after t hours, is given by: $\frac{dA}{dt} = \frac{1}{8}\pi t$.

Initially the disc has a radius of 2 metres.
Which of the following is the correct expression for the area after t hours?

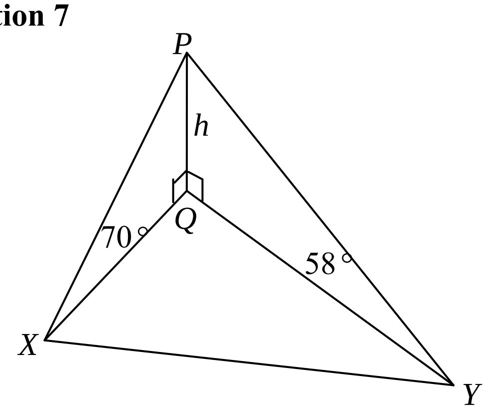
(A) $A = \frac{1}{8}\pi t^2$ (B) $A = \frac{1}{16}\pi t^2$ (C) $A = \frac{1}{8}\pi t^2 + 4\pi$ (D) $A = \frac{1}{16}\pi t^2 + 4\pi$

Question 6

How many distinct permutations of the letters of the word 'DIVIDE' are possible in a straight line when the word begins and ends with the letter D?

(A) 12 (B) 180 (C) 360 (D) 720

Question 7



In the diagram shown, $XQ =$

(A) $h\cos 70°$ (B) $h\tan 70°$ (C) $h\cot 100°$ (D) $h\cot 70°$

Question 8

$$\sum_{n=1}^{k+1} \frac{n}{2}(n+1) =$$

(A) $\frac{k+1}{2}(k+2)$

(B) $\sum_{n=1}^{k} \frac{n}{2}(n+1) + \frac{k+1}{2}(k+2)$

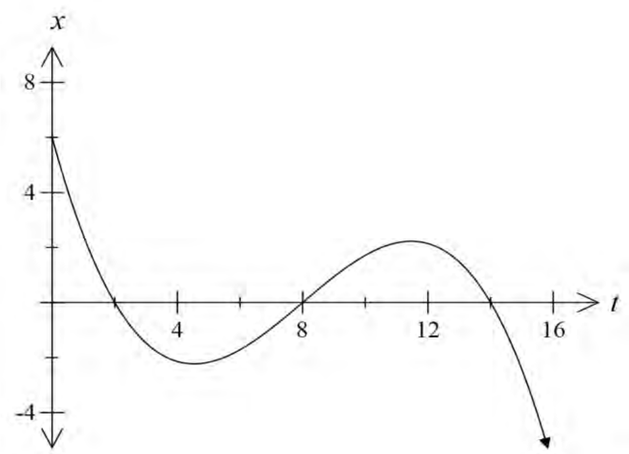
(C) $\frac{k}{2}(k+1) + \frac{k+1}{2}(k+2)$

(D) $\sum_{n=1}^{k} \frac{n}{2}(n+1) + 1 + 3 + 6 + \ldots + \frac{k+1}{2}(k+2)$

Question 9

The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph.

When was the particle moving with greatest speed?



(A) $t=0$ (B) $t=4.5$ (C) $t=8$ (D) $t=11.5$

Question 10

The interval joining the points $A(1,3)$ and $B(a,b)$ is divided internally in the ratio 2:3 by the point $(3,13)$. What are the values of a and b?

(A) $a=6$ and $b=28$ (B) $a=6$ and $b=37$

(C) $a=9$ and $b=28$ (D) $a=9$ and $b=37$

End of Section A

SECTION B

Question 11 (15 marks) **Use a SEPARATE writing booklet**

Marks

a) Find the acute angle between the lines $2x - y - 1 = 0$ and $\frac{1}{4}x - y + 1 = 0$. **2**

Give your answer to the nearest degree.

b) If A is the point $(-2,-1)$ and B is the point $(1,5)$, find the coordinates of the point P which divides the interval AB externally in the ratio $2:5$. **2**

c) The environment committee needs to seat 10 of its members (5 females and 5 males) at a round table.

(i) How many different seating arrangements are possible, without restrictions? **1**

(ii) What is the probability that two particular males are to be seated next to each other? **2**

d) A team of 5 men and 4 women is to be chosen at random from a group of 8 male and 7 female mathematicians. If Rodney and Deborah are both hoping to be chosen, what is the probability that:

(i) Both are chosen? **2**

(ii) Neither is chosen? **1**

e) Solve $\frac{4+x}{2x} > 1$ **2**

f) The polynomial $p(x)$ is given by $p(x) = x^3 + bx^2 + cx - 10$ where b and c are constants. The three zeroes of $p(x)$ are -1, 2 and α.

(i) Find the values of b and c **2**

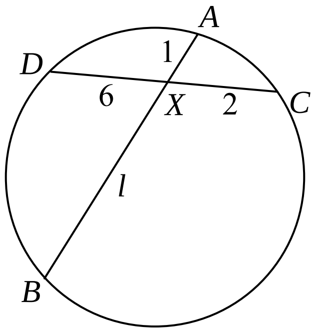
(ii) Hence or otherwise find the value of α **1**

End of Question 11

Question 12 (15 marks) **<u>Use a SEPARATE writing booklet</u>**

Marks

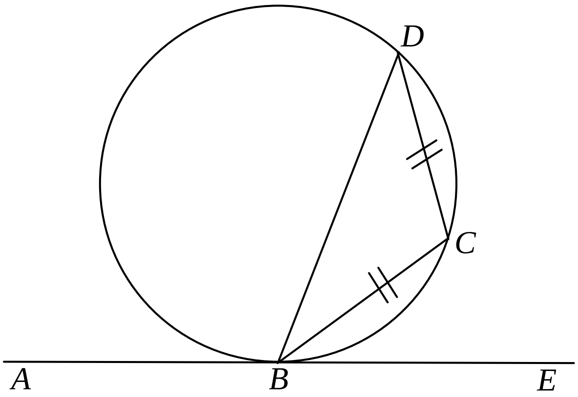
a)



NOT TO SCALE

In the diagram above, find the length, l. Justify your working with reasons as appropriate. **2**

b) In the following diagram, $BC = DC$. AE is a tangent to the circle.



(i) Why is $\angle ABD = \angle BCD$? **1**

(ii) Prove that BC bisects $\angle DBE$. **2**

c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ with O the vertex.

(i) Show that the gradient of the chord PQ is $\dfrac{p+q}{2}$. **1**

(ii) What are the co-ordinates of the point where the chord PQ passes through the y-axis? **2**

1

(iii) Find the gradient of OP . **1**

Question 12 continues on page 7

Question 12 continued

(iv) If $OP \perp OQ$, show that $pq = -4$. **1 mark**

(v) Given that the gradient of the tangents to the parabola at P and Q are p and q respectively, find the equation of the locus of T, the point where the tangents intersect. **2 marks**

You may assume that the equation of the tangent at the point P is given by $y = px - ap^2$ **(DO NOT PROVE THIS)**

d) Prove by mathematical induction that $\left(3^{2n} - 1\right)$ is divisible by 8 **3 marks**

End of Question 12

Question 13 (15 marks) **Use a SEPARATE writing booklet**

Marks

a) Solve, in radians, for $0 \le x \le 2\pi$, **2**

$\sin 2x - \cos x = 0$

b) Find $\int \frac{dx}{5+x^2}$ **1**

c) Consider the curve $y = 2\cos^{-1}\frac{x}{3}$

(i) Find the range, and hence sketch the curve $y = 2\cos^{-1}\frac{x}{3}$ **2**

(ii) Find the gradient of the tangent to the curve, at the point where $x = \frac{3\sqrt{3}}{2}$. **2**

d) (i) Write the expansion for $\tan(\alpha - \beta)$ **1**

(ii) Hence, find x so that $\tan^{-1} x = \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right)$ **2**

e) Consider the curve $f(x) = (x-2)^2$

(i) If the domain is to be restricted to the largest possible domain that contains $x = 0$, so that an inverse function will exist, state the domain. **1**

(ii) What is the domain of $y = f^{-1}(x)$? **1**

(iii) What is the equation of $y = f^{-1}(x)$? **1**

(iv) Explain why $x = (x-2)^2$ gives the points of intersection of $y = f(x)$ and $y = f^{-1}(x)$ and hence why $x = 1$ is the only point of intersection **2**

End of Question 13

Question 14 (15 marks) **<u>Use a SEPARATE writing booklet</u>**

Marks

a) (i) Show that $\frac{u}{u+1} = 1 - \frac{1}{u+1}$ **1**

(ii) Hence, find $\int \frac{1}{1+\sqrt{x}}\, dx$ using the substitution $\left(x = u^2,\, u \geq 0\right)$ **3**

b) At 7 pm on a Wednesday evening, Mr Huxley's water tank was full.

The capacity of the tank was 3 000 litres.

Unfortunately, the tap on the tank was leaking in such a way that the change in volume at any time (t) hours was proportional to the volume (V) of the tank.

This means that $\frac{dV}{dt} = -kV$.

(i) Show that $V = V_0 e^{-kt}$ is a solution of this equation. **1**

(ii) Given that the volume of the tank after 3 hours is 1900 litres, show that $k = 0.1523$ correct to 4 decimal places. **2**

(iii) By the time Mr Huxley discovered that the tank was leaking, there were only 250 litres of water remaining. **2**

At what time and on which day did Mr Huxley discover the leak

(Answer correct to the nearest minute)?

Question 14 continues on page 10

Question 14 continued

Marks

c) A particle moves along a straight line about a fixed point *O* so that its acceleration, a ms^{-2}, at time t seconds is given by $a = 4\cos\left(2t + \frac{\pi}{6}\right)$.

Initially the particle is moving to the right with a velocity of 1 ms^{-1} from a position $\frac{\sqrt{3}}{2}$ metres to the left of *O*.

(i) Find an expression for the velocity of the particle after t seconds. **2**

(ii) Find an expression for the position of the particle after t seconds. **2**

(iii) Show that the particle changes directions when $t = \frac{5\pi}{12}$ seconds. **1**

(iv) At what time does the particle return to its initial position for the first time? **1**

End of Question 14

End of Examination

SECTION A ANSWER SHEET

- **Detach this sheet** and use it to mark the answers to the questions in Section A
- Mark the answer by shading the letter that matches with the correct answer
- If you make a mistake, draw a cross through the incorrect answer

Name: ______________________________ **Class:** ________________

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Extension 1 Trial HSC Multiple Choice

Question 1: **C** $3\cdot 6=10e^{-5k}$. $\ln(0\cdot 36)=-5k \Rightarrow k\approx 0\cdot 204$

Question 2: **A** $P(2)=0$ $P(2)=30+a \Rightarrow a=-30$

Question 3: **A** The range of $\tan^{-1}\theta$ is $\frac{-\pi}{2}<\theta<\frac{\pi}{2}$. Quadrant 2. Related angle $\frac{\pi}{4}$. $\therefore$ $\frac{-\pi}{4}$

Question 4: **D** $\text{QT}\times\text{PT}=x^2$ i.e. $12\times 8=x^2$ and $x=4\sqrt{6}$

Question 5: **D** $A=\int\frac{1}{8}\pi t.$ $A=\frac{1}{16}\pi t^2+C.$ $\pi\times 2^2=\frac{1}{16}\pi\times 0^2+C.$ $\therefore A=A=\frac{1}{16}\pi t^2+4\pi$

Question 6: **A** Place the D's. There are 4 letters left, including 2 "I's". $\frac{4!}{2!}=12$

Question 7: **D** In right triangle XQP, $\tan(70^0)=\frac{h}{XQ}\Rightarrow XQ=h\cot(70^0)$

Question 8: **B** Sum to the *k*th term + the *(k+1)* term.

Question 9: **A** Velocity $=\left|\frac{dx}{dt}\right|$. $\frac{dx}{dt}$ is the gradient of the tangent. Steepest at $t=0$

Question 10: **A** Using your formula: $(3,13)=\left(\frac{3\times 1+2a}{3+2},\frac{3\times 3+2b}{3+2}\right)\Rightarrow(a,b)=(6,28)$

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE3 solves problems involving permutations and *combinations*, *inequalities*, *polynomials*, circle geometry and parametric representations

H5 applies appropriate techniques from the study of calculus, *geometry*, *probability*, trigonometry and series to solve problems

Outcome	Solutions	Marking Guidelines
H5	**(a)** $y = 2x - 1,\ m_1 = 2$ $y = \frac{1}{4}x + 1,\ m_2 = \frac{1}{4}$ $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ $\tan\theta = \frac{2 - \frac{1}{4}}{1 + 2\left(\frac{1}{4}\right)}$ $\tan\theta = \frac{7}{6}$ $\therefore \theta = 49°$ (nearest degree)	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
H5	**(b)** $A(-2,-1),\ B(1,5),\ P(x,y)$ Externally in the ratio $-2:5$ $x_P = \frac{mx_2 + nx_1}{m+n}$ $y_P = \frac{my_2 + ny_1}{m+n}$ $x_P = \frac{(-2)(1)+(5)(-2)}{-2+5}$ $y_P = \frac{(-2)(5)+(5)(-1)}{-2+5}$ $x_P = \frac{-2-10}{3}$ $y_P = \frac{-10-5}{3}$ $x_P = -4$ $y_P = -5$ $\therefore$ The coordinates of P are $(-4,-5)$.	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
PE3, H5	**(c) (i)** $(n-1)! = 9!$ $= 362880$	**1 mark** Correct solution.
	(c) (ii) $2 \times 8! = 80640$ $P(E) = \frac{80640}{362880}$ $= \frac{2}{9}$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.
PE3, H5	**(d) (i)** No restrictions: ${}^8C_5 \times {}^7C_4 = 1960$ Both chosen: ${}^7C_4 \times {}^6C_3 = 700$ $\therefore$ The probability that both are chosen $= \frac{700}{1960}$ $= \frac{5}{14}$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.

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(d) (ii)

Neither is chosen: ${}^{7}C_5 \times {}^{6}C_4 = 315$

$\therefore$ The probability that either are chosen $= \dfrac{315}{1960}$

$= \dfrac{9}{56}$

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1 mark
Correct solution.

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PE3

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(e)

$\dfrac{4+x}{2x} > 1$

$\dfrac{(4+x)x^2}{2x} > 1.x^2$

$\dfrac{(4+x)x}{2} > x^2$

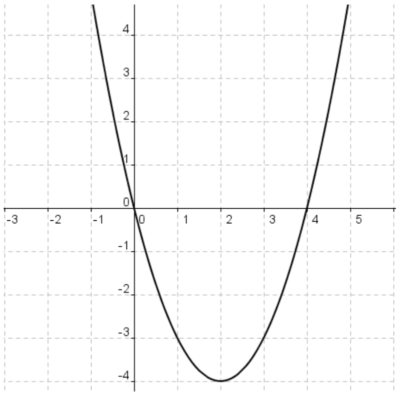
$4x + x^2 > 2x^2$

$0 > x^2 - 4x$

$x^2 - 4x < 0$

$x(x-4) < 0$

$\therefore 0 < x < 4$



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2 marks
Correct solution.
1 mark
Substantial progress towards correct solution.

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PE3

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(f) (i)

$p(x) = x^3 + bx^2 + cx - 10$

$p(-1) = 0$

$(-1)^3 + b(-1)^2 + c(-1) - 10 = 0$

$b - c - 11 = 0$

$b = 11 + c \quad ...(1)$

$p(2) = 0$

$(2)^3 + b(2)^2 + c(2) - 10 = 0$

$4b + 2c - 2 = 0$

$2b + c - 2 = 0 \quad ...(2)$

Substitute (1) into (2) and solve for *c*.

$2(11+c) + 2c - 2 = 0$

$22 + 2c + 2c - 2 = 0$

$\therefore c = -7$

$\therefore b = 11 + (-7)$

$\therefore b = 4$

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2 marks
Correct solution.
1 mark
Substantial progress towards correct solution.

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(f) (ii)

$\alpha + \beta + \delta = \dfrac{-b}{a}$

$\alpha - 1 + 2 = -4$

$\alpha + 1 = -4$

$\therefore \alpha = -5$

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1 mark
Correct solution.

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Year 12 Mathematics Extension 1 Trial HSC Examination 2013

Question No. 12 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE2 uses inductive reasoning in the construction of proofs

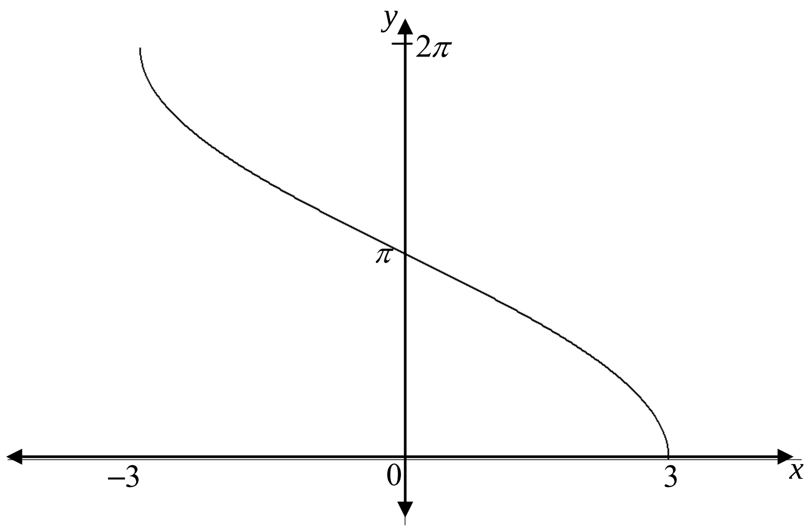
PE3 solves problems involving permutations and combinations, inequalities, polynomials, **circle geometry and parametric representations**

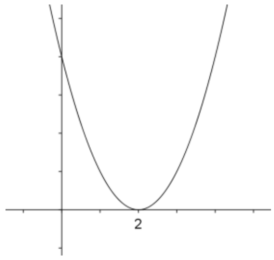
Outcome	Solutions	Marking Guidelines
PE3	**(a)** $AX.XB = CX.XD$ (product of intercepts on intersecting chords are equal) $1.l = 2.6$ $\therefore\ l = 12$	**2 marks** Correct solution including reason. **1 mark** Demonstrates knowledge of appropriate circle property.
PE3	**(b) (i)** Angle between a chord and a tangent is equal to the angle in the alternate segment.	**1 mark** Demonstrates knowledge of appropriate circle property.
PE3	**(ii)** Let $\angle DBC = \alpha$ $\therefore \angle BDC = \alpha$ (angles opposite equal sides in isosceles ΔBCD) $\angle BDC + \alpha + \alpha = 180°$ (angle sum of ΔBCD) $\angle BDC = 180 - 2\alpha$ Now, $\angle ABD = \angle BDC$ (alternate segment theorem) $= 180 - 2\alpha$ $\angle ABD + \angle DBC + \angle CBE = 180°$ (angles on a straight line) $180 - 2\alpha + \alpha + \angle CBE = 180°$ $\therefore \angle CBE = \alpha$ $\angle DBC = \angle CBE = \alpha$ $\therefore BC$ bisects $\angle DBE$	**2 marks** Correct solution including full reasoning. **1 marks** Substantial progress towards correct solution OR correct working with insufficient reasoning.
PE3	**(c) (i)** $m_{PQ} = \dfrac{aq^2 - ap^2}{2aq - 2ap}$ $= \dfrac{a(q^2 - p^2)}{2a(q-p)}$ $= \dfrac{a(q+p)(q-p)}{2a(q-p)}$ $= \dfrac{p+q}{2}$	**1 mark** Correct solution.
PE3	**(ii)** Equation of PQ $y - y_1 = m(x - x_1)$ $y - ap^2 = \dfrac{p+q}{2}(x - 2ap)$ If $x = 0$, $y - ap^2 = \dfrac{p+q}{2} \times -2ap$ $= -ap^2 - apq$ $y = -apq$ ie. passes through the y-axis at $-apq$	**2 marks** Correct solution. **1 mark** Substantial progress towards full solution.
PE3	**(iii)** $m_{OP} = \dfrac{ap^2}{2ap}$ $= \dfrac{p}{2}$	**1 mark** Correct solution.

PE3	**(iv)** Similarly, $m_{OQ} = \frac{q}{2}$ (Continued next page) If $OP \perp OQ$, $m_{OP}.m_{OQ} = -1$ $\frac{p}{2}.\frac{q}{2} = -1$ $\therefore pq = -4$	**1 mark** Correct solution.
PE3	**(v)** Equation of tangent at P: $y = px - ap^2$ Equation of tangent at Q: $y = qx - aq^2$ Point of intersection: $px - ap^2 = qx - aq^2$ $px - qx = ap^2 - aq^2$ $(p-q)x = a(p^2 - q^2)$ $x = \frac{a(p+q)(p-q)}{p-q}$ $= a(p+q)$ When $x = a(p+q)$, $y = ap(p+q) - ap^2$ $= ap^2 + apq - ap^2$ $= apq$ But, $pq = -4$ $\therefore y = -4a$ ie. Locus of T is a horizontal line with equation $y = -4a$	**2 marks** Correct solution. **1 mark** Substantial progress towards full solution.
HE2	**(d)** To prove $3^{2n} - 1$ is divisible by 8 1. Prove true for $n = 1$ $3^{2.1} - 1 = 8$ which is divisible by 8 $\therefore$ True for $n = 1$. 2. Assume true for $n = k$ ie. assume $3^{2k} - 1 = 8M$ where M is a positive integer Prove true for $n = k+1$ ie. prove $3^{2(k+1)} - 1$ is divisible by 8 $3^{2(k+1)} - 1 = 3^{2k+2} - 1$ $= 3^{2k}.3^2 - 1$ $= (8M+1).9 - 1$ (since $3^{2k} = 8M + 1$, from assumption) $= 72M + 9 - 1$ $= 72M + 8$ $= 8(9M+1)$ which is divisible by 8 $\therefore$ True for $n = k+1$ 3. If the result is true for $n = k$, it is also true for $n = k+1$. Since the result is true for $n = 1$, it is then also true for $n = 1+1 = 2$, $n = 2+1 = 3$, etc. $\therefore$ The result is true for all positive integral values of n.	**3 marks** Correct solution showing full reasoning. **2 marks** Substantial progress towards full solution including proof for n = 1. **1 mark** Proves result is true for n = 1.

Outcomes Addressed in this Question

HE4 Uses the relationship between functions, inverse functions and their derivatives
HE7 Evaluates mathematical solutions to problems and communicates them in an appropriate form

Outcome	Solutions	Marking Guidelines
HE7	(a) $\sin 2x - \cos x = 0$ $2\sin x\cos x - \cos x = 0$ $\cos x(2\sin x - 1) = 0$ $\cos x = 0$ or $2\sin x - 1 = 0$ $\sin x = \frac{1}{2}$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ (from the graph) or $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ $\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$.	2 marks : correct answer 1 mark : significant progress towards answer
H5	(b) $\int \frac{dx}{5+x^2} = \int \frac{1}{(\sqrt{5})^2 + x^2}dx$ $= \frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + c$	1 mark : correct answer
HE4	(c) (i) Range of $y = \cos^{-1}\frac{x}{3}$ is $0 \le y \le \pi$ $\therefore$ for $y = 2\cos^{-1}\frac{x}{3}$, range is $0 \le y \le 2\pi$. Domain of $y = \cos^{-1}\frac{x}{3}$ is $-1 \le 3x \le 1$, $\therefore$ domain of $y = 2\cos^{-1}\frac{x}{3}$ is $-3 \le x \le 3$. 	2 marks : correct range and correct graph 1 mark: correct range but incorrect graph or correct graph for incorrect range

HE4	(ii) $y = 2\cos^{-1}\frac{x}{3}$ $y' = -2.\frac{\frac{d}{dx}\left(\frac{x}{3}\right)}{\sqrt{1-\left(\frac{x}{3}\right)^2}}$ since $\frac{d}{dx}\left(\cos^{-1} f(x)\right) = \frac{-f'(x)}{\sqrt{1-\left(f(x)\right)^2}}$ $= \frac{\frac{-2}{3}}{\sqrt{\frac{9-x^2}{9}}}$ $= \frac{-2}{\sqrt{9-x^2}}.$ When $x = \frac{3\sqrt{3}}{2}$, $y' = \frac{-2}{\sqrt{9-\frac{27}{4}}} = \frac{-4}{3}$ $\therefore$ gradient of tangent at $x = \frac{3\sqrt{3}}{2}$ is $\frac{-4}{3}$.	2 marks : correct derivative and answer 1 mark : significant progress towards answer
	(d) (i) $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$ (ii) Let $\alpha = \tan^{-1}\frac{1}{2}$ and $\beta = \tan^{-1}\frac{1}{3}$. $\therefore \tan\alpha = \frac{1}{2}$, $\tan\beta = \frac{1}{3}$ Using result in (i), $\tan(\alpha-\beta) = \frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{2}\times\frac{1}{3}}$ $\therefore \tan(\alpha-\beta) = \frac{1}{7}$ $\therefore \alpha-\beta = \tan^{-1}\left(\frac{1}{7}\right)$ $\therefore \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right)$ $\therefore x = \frac{1}{7}$.	1 mark: correct result 2 marks : correct solution 1 mark : significant progress towards answer
HE4	(e)  (i) Will have an inverse if strictly increasing or strictly decreasing only. Largest domain, containing $x = 0$ where this occurs is $x \leq 2$	1 mark: correct answer

HE4	(ii) Domain of $y=f^{-1}(x)$ is the range of $y=f(x)$. Range of $y=f(x)$ is $y\ge 0$. $\therefore$ domain of $y=f^{-1}(x)$ is $x\ge 0$.	1 mark: correct answer
HE4	(iii) Interchanging *x* and *y*, the inverse is $x=(y-2)^2$ $y-2=\pm\sqrt{x}$ $y=2\pm\sqrt{x}$ But as $x\le 2$ for the inverse to exist, $y=2-\sqrt{x}$.	1 mark: correct answer
HE4	(iv) $y=f(x)$ and $y=f^{-1}(x)$ **intersect on the line *y* = *x*.** $\therefore$ $y=(x-2)^2$ and $y=x$ can be solved simultaneously to give the points of intersection for $y=f(x)$ and $y=f^{-1}(x)$. They meet when $x=(x-2)^2$. i.e. when $x=x^2-4x+4$ $x^2-5x+4=0$ $(x-4)(x-1)=0$ $x=1$ or 4 But as $x\le 2$ for the inverse to exist, $y=f(x)$ and its inverse meet when $x=1$ only.	2 marks: correct explanation for why $x=(x-2)^2$ gives the point of intersection; and correctly solves equation and explains why one solution only. 1 mark: one of above

Outcomes Addressed in this Question

HE6	determines integrals by reduction to a standard form through a given substitution
HE3	uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay
H4	expresses practical problems in mathematical terms based on simple given models

Outcome	Solutions	Marking Guidelines
(a) (i)	$\text{LHS} = \dfrac{u}{u+1}$ $\text{RHS} = 1 - \dfrac{1}{u+1}$ $= \dfrac{u+1-1}{u+1}$ $= \dfrac{u}{u+1} = \text{LHS}$	**1 mark** ~ correct solution
(ii) HE6	$\int \dfrac{1}{1+\sqrt{x}}\,dx \qquad x = u^2 \Rightarrow dx = 2udu$ $= \int \dfrac{1}{1+u} 2u\,du$ $= 2\int \dfrac{u}{u+1}\,du$ $= 2\int \left(1 - \dfrac{1}{u+1}\right) du$ $= 2\left(u - \ln\left(u+1\right)\right) + c$ $= 2\left(\sqrt{x} - \ln\left(\sqrt{x}+1\right)\right) + c$	**3 marks** ~ correct solution **2 marks** ~ substantial progress towards solution **1 mark** ~ limited progress towards solution
(b) (i) HE3	$V = V_0 e^{-kt}$ $\dfrac{dV}{dt} = V_0 \times -ke^{-kt}$ $= -kV_0 e^{-kt}$ $= -kV$ $\therefore$ Solution to the given equation.	**1 mark** ~ correct solution
(ii) HE3	$t = 3, V = 1900$ From given information, $V_0 = 3000$ $1900 = 3000e^{-3k}$ $e^{-3k} = \dfrac{1900}{3000} = \dfrac{19}{30}$ $\therefore e^{3k} = \dfrac{30}{19}$ $\therefore k = \ln\left(\dfrac{30}{19}\right) \approx 0.1523$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution

(iii) HE3	$V=250$ $250=3000e^{-\frac{1}{3}\ln\left(\frac{30}{19}\right).t}$ $e^{-\frac{1}{3}\ln\left(\frac{30}{19}\right).t}=\frac{250}{3000}$ $e^{\frac{1}{3}\ln\left(\frac{30}{19}\right).t}=\frac{3000}{250}=12$ $\frac{1}{3}\ln\left(\frac{30}{19}\right).t=\ln 12$ $t=\frac{\ln 12}{\frac{1}{3}\ln\left(\frac{30}{19}\right)}=16.3209257\text{ hours}\approx 16\text{ hours }19\text{ minutes}$ $\therefore$ Mr Huxley discovered the leak on Thursday @ 11:19 am	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(c) (i) H4	$a=4\cos\left(2t+\frac{\pi}{6}\right)$ $v=2\sin\left(2t+\frac{\pi}{6}\right)+c$ $t=0, v=1$ $1=2\sin\left(2.0+\frac{\pi}{6}\right)+c$ $1=2.\frac{1}{2}+c$ $\therefore c=0$ $\therefore v=2\sin\left(2t+\frac{\pi}{6}\right)$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(ii) H4	$x=-\cos\left(2t+\frac{\pi}{6}\right)+k$ $t=0,\ x=-\frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2}=-\cos\left(\frac{\pi}{6}\right)+k$ $k=0$ $\therefore x=-\cos\left(2t+\frac{\pi}{6}\right)$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution

(iii) H4	$v=0,\ 2\sin\left(2t+\frac{\pi}{6}\right)=0$ $2t+\frac{\pi}{6}=0,\ \pi,\ 2\pi,......$ $2t=-\frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{11\pi}{6},.....$ $t=\frac{5\pi}{12},\ \frac{11\pi}{12},.....\quad (t>0)$ Since $v=0$ when $t=\frac{5\pi}{12}$, particle changes direction.	**1 mark** ~ correct solution
(iv) H4	$x=-\frac{\sqrt{3}}{2}$ $-\cos\left(2t+\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}$ $\cos\left(2t+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $2t+\frac{\pi}{6}=\frac{\pi}{6},\ \frac{11\pi}{6},\ \frac{13\pi}{6},\ \frac{23\pi}{6},......$ $2t=0,\ \frac{5\pi}{3},\ 2\pi,\ \frac{11\pi}{3},......$ $t=0,\ \frac{5\pi}{6},\ \pi,\ \frac{11\pi}{6},......$ Returns to starting position again when $t=\frac{5\pi}{6}$.	**1 mark** ~ correct solution